MATHEMATICS

The development of mathematical knowledge and skills contributes to children's ability to make sense of the world and to solve mathematical situations they encounter in their everyday lives. Knowledge of basic math concepts and the skill to use math operations to solve mathematical situations are fundamental aspects of school readiness and are predictive of later success in school and in life. The components within this domain address number sense and quantity; number relationships and operations; classification and patterning; measurement, comparison, and ordering; and geometry and spatial sense.

Children with disabilities may demonstrate alternate ways of meeting the goals of mathematics development. For example, a child who is blind may begin to identify braille numbers and a child with a physical disability may identify numerals through use of an eye gaze. Children with disabilities may reach many of these same goals, but at a different pace, with a different degree of accomplishment, and in a different order than their peers. However, the goals for all children are the same, even though the path and the pace toward realizing the goals may be different. Principles of universal design for learning (UDL) offer the least restrictive and most inclusive approach to developing environments and curricula that best serve the mathematics development of all children.

Remember: While this domain represents general expectations for mathematics development, each child will reach the individual standards at their own pace and in their own way.

M 1:   Number Sense and Quantity

M 2:   Number Relationships and Operations

M 3:   Classification and Patterning

M 4:   Measurement, Comparison, and Ordering

M 5:   Geometry and Spatial Sense

MATHEMATICS

Component 1: Number Sense and Quantity

Standard 1.a: Children develop number recognition and counting skills and learn the relationship between numbers and the quantity they represent.

By the following age ranges, children typically, for example:

0-9m
› Hold two objects, one in each hand
› Gesture (e.g., for "more" when eating)

9-18m
› Use words, gestures, and/ or signs for action phrases (e.g., "all gone" and "more")
› Recognize that there are "one" or "two" of something

18-24m
› Name their first number word—typically "two" (or they hold up two fingers)
› Bring 2 treats when asked to get treats for two people
› Match small and large objects (e.g., counting bears to make "Mommy and Me" pairs)

24-36m
› Verbally count to 10 (or in some way indicate a knowledge of words for the numbers from one to ten in sequence) with occasional errors
› Count up to 5 objects accurately
› Name and identify some written numerals
› Identify quantities associated with some numbers

36-48m
› Quickly name the number in a group of objects, up to four, without counting
› Verbally count to 20 (or in some way indicate knowledge of the words for the numbers from 1 to 20 in sequence) with occasional errors
› Use strategies to accurately count sets of up to 10 objects
› Understand that the last number counted represents the number of objects in a set
› Associate a quantity with a written numeral up to five
› Recognize and write some numerals up to 10

48-60m
› Quickly name the number in a group of objects, up to 10
› Verbally count beyond 20 (or in some way indicate knowledge of numbers beyond 20 in sequence), demonstrating an understanding of the number pattern
› Use strategies to count large sets of objects (more than 10)
› Know the number that comes before or after a specified number (up to 20)
› Recognize and order each written numeral up to 10
› Associate a quantity with a written numeral up to 10

> "Young children have the capacity and interest to learn meaningful mathematics. Learning such mathematics enriches their current intellectual and social experiences and lays the foundation for future learning."
> - National Research Council, 2010

Component 2: Number Relationships and Operations

Standard 2.a: Children learn to use numbers to compare quantities and solve mathematical situations.

By the following age ranges, children typically, for example:

0-9m
› Hold two objects, one in each hand
› Explore their fingers and toes

9-18m
› Demonstrate early one-to-one correspondence (e.g., filling containers with objects by dropping them in one at a time)
› In most instances, choose a set that has more of something they prefer over a set that has less, when given the option
› Create larger and smaller sets of objects by grouping and ungrouping items (e.g., placing and removing rings on a vertical peg)

18-24m
› Begin to say or gesture the number "two" when asked how old they are
› Put objects in accurate, one-to-one correspondence when supported by the context (e.g., placing one plastic egg into each indentation of an egg carton)
› Compare collections that are quite different in size (e.g., one that is at least twice the other)
› Notice when another child has more of something and gesture or verbalize "want more"
› Put groups of objects together and begin to subtract (e.g., share)

24-36m
› Use visual cues to approximate which of 2 sets of objects has more
› Understand that putting 2 sets of objects together makes "more" and taking sets of objects apart will make less
› Add and subtract with sets of objects smaller than three

36-48m
› Understand that a whole is a larger quantity than its parts (e.g., when looking at 3 nests with 3 eggs in each, says a big number such as 8 or 10 to describe how many eggs there are)
› Use toys and other objects as tools to solve simple addition and subtraction problems when the total is smaller than five
› Use one-to-one correspondence to compare small sets of similar objects

48-60m
› Use counting to compare 2 sets of objects and to determine which set has more, less, or the same than the other
› Understand that adding one or taking away one changes the number in a group of objects by exactly one
› Use toys and other objects as tools to solve simple addition and subtraction problems with totals smaller than 10

Component 3: Classification and Patterning

Standard 3.a: Children learn to order and sort objects by common attributes, to identify patterns, and to predict the next sequence in a pattern.

By the following age ranges, children typically, for example:

0-9m
- Classify informally as they intuitively recognize objects or situations as similar (e.g., "things I can hold")
- Explore the size and shape of objects through various means (banging, mouthing, dropping, etc.)
- Engage in repeated actions, including movements and vocalizations
- Follow daily routines (e.g., patterns), such as being fed and then going to sleep almost immediately
- Enjoy and begin to anticipate repetition, such as playing a "peek-a-boo" game or hearing a familiar song each time they are diapered

9-18m
- Identify objects or creatures by implicitly recognizing their basic attributes and applying labels that adults perceive as classes (e.g., all canines are "doggies"; all felines are "kitties")
- Form sets intuitively in which objects in each set are the same (identical) and objects in the other set are different (e.g., putting all of the dolls in one pile and all of the cars in another)
- Repeat certain action sequences over and over, such as filling and emptying containers
- Make patterns intuitively by repeating particular movements and vocalizations
- Watch, bounce, or clap to rhythmic sounds or sing along when an adult sings a song or chant

18-24m
- Sort and match toys and other objects by attributes, such as color or size
- Form sets with objects that are similar in some properties (but not necessarily identical), such as separating groups by color
- Make simple patterns of movement through repeated rhythmic activity
- Copy an adult's made-up verbal pattern (e.g., "me, me, moo")
- Recognize and begin to participate in patterns within stories and in songs

24-36m
- Follow a verbal rule for sorting objects into sets
- Notice when two things share similar attributes
- Recognize and extend a simple repeating pattern (e.g., stomp-clap-stomp-clap), with modeling and support

36-48m
- Sort objects by one attribute into two or more groups (e.g., size: big, medium, and small)
- Classify everyday objects that go together (e.g., mittens, hats, coats)
- Demonstrate recognition of a simple, repeating pattern
- Replicate, complete, and extend repeating patterns
- Recognize, name, and extend basic growing (or enlarging) patterns (e.g., "one more")

48-60m
- Sort objects by more than one attribute (e.g., color and shape) into two or more groups
- Sort sets of objects by one characteristic, then sort by a different characteristic and explain the sorting rules (e.g., "These are all of the red ones, but these are all of the big ones")
- Extend sequential patterns and replicate these patterns using different materials or modes (e.g., on being told a pattern, replicating the pattern with manipulatives)
- Identify the core unit of sequentially repeating patterns (e.g., that set of characteristics or items that repeat)
- Replicate and extend simple growing (or enlarging) patterns

Component 4: Measurement, Comparison, and Ordering

Standard 4.a: Children learn to measure objects by their various attributes to make comparisons.

By the following age ranges, children typically, for example:

0-9m

› Explore the size and shape of objects through various means (banging, mouthing, dropping, etc.)

› Explore volume as they wrap their fingers around an object or around an adult's finger

› Explore weight as they pull a toy toward themselves

› Explore speed by moving hands or legs

9-18m

› Notice large differences in size between two objects (e.g., pointing to the bigger ball)

› Begin to use such words as "big" and "more" to describe sizes or amounts

› Explore relative size by trying to squeeze a large object into a smaller container (e.g., putting a doll into doll stroller and then trying to fit themselves into the stroller)

18-24m

› Use words such as "big," "small," and "more" to indicate differences in quantity

› Understand and use general measurement words, such as "big" and "hot"

› Recognize when their food bowl is empty and gesture to indicate that fact, or say "more" or "all gone"

› Find and point to small objects (e.g., the tiny mouse on the pages of *Goodnight Moon*)

24-36m

› Compare small quantities (e.g., knowing that "two" is more than "one" or choosing the larger bowl for cereal over the smaller one)

› Know the sequence of some parts of their daily routine

› Use language to compare the sizes of objects (e.g., "big" and "little"; and "mommy," "daddy," and "baby")

36-48m

› Compare two small sets of objects (five or fewer)

› Make small series of objects (e.g., putting three or four objects in order by length)

› Recognize differences in measurable attributes by direct comparison measuring (e.g., when trying to pour the same amount of juice into three cups, looking to see if one cup has more than the others)

› Use multiple copies of the same unit to measure (e.g., seeing how many "building blocks high" a pillow fort is)

› Use comparative language (e.g., "shortest," "heavier," "biggest")

48-60m

› Order (or seriate) four or more items by decreasing or increasing a relative attribute when differences are perceptually clear (e.g., arranging a rock collection from the largest to the smallest)

› Use some appropriate tools to measure different attributes (e.g., choosing a scale for weight and a cup for volume)

› Use measurement language to describe the attributes of objects (e.g., "This is three-blocks long.")

> "Infants' and toddlers' natural curiosity initially sparks their interest in understanding the world from a mathematical perspective, and the adults and communities that educate and care for them also provide experiences that serve as the basis for further mathematics learning.
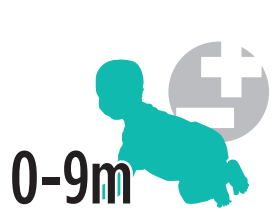>
> - National Research Council, 2010

Component 5: Geometry and Spatial Sense

Standard 5.a: Children learn to identify shapes and their attributes, solve mathematical situations using shapes, and explore the positions of objects in space.
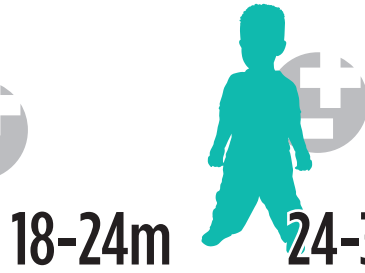
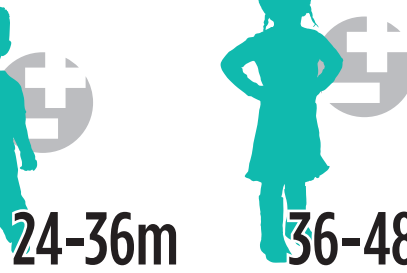## By the following age ranges, children typically, for example:

0-9m

› Explore the size and shape of objects through various means (e.g., banging, mouthing, dropping, etc.)

› Explore the way objects move by tracking objects with their eyes and head

› Explore distance by reaching for something

› Explore their spatial sense through movement, both involuntary and voluntary (e.g., being picked up, scooting, and pulling up)

9-18m

› Explore how differently shaped objects fit or do not fit together by manipulating such things as nesting cups or stacking cones

› Explore barriers to movement when not able to walk or push past something

› Compare the attributes of objects or pictures and identify those that are similar

› Explore their spatial sense (e.g., by bumping into things; squeezing into a tight space; or looking at an adult or a toy from a different angle, when bending over, or with head turned)

18-24m

› Match familiar shapes (e.g., circle, square, triangle) that have the same size and the same orientation

› Attempt to stack blocks as high or higher than themselves

› Match simple shapes (e.g., placing a shape on a shape board)

› Explore gravity (e.g., push toy cars down an incline, such as a slanted board)

› Respond to spatial directions, such as "come here," "go over there," and "get down on the floor," especially if the words are accompanied by gestures, such as pointing and/or signs

24-36m

› Link or sort (put together) familiar shapes (e.g., circle, square, triangle) that have different sizes and orientation

› Build pictures or designs with two-dimensional shapes and create buildings or structures with three-dimensional shapes, such as building blocks

› Recognize and name some familiar shapes in their home language(s)

› Understand and use basic language related to locations (e.g., "above," "below," "under," "over")

36-48m

› Build familiar two-dimensional shapes from components or parts (e.g., using a set of circle, rectangle, and line shapes to create an image of a snowman)

› Combine and separate shapes to make designs or pictures (e.g., completing shape puzzles)

› Build simple examples of buildings, structures, or areas (e.g., their classroom or playground) with three-dimensional shapes, such as building blocks

› Name familiar two-dimensional shapes in their home language(s) (circle, triangle, square, rectangle), regardless of their size or orientation

› Use basic language to describe their location (e.g., "I am under the bed.)

› Correctly follow directions involving their own positions in space (e.g., "move forward," "sit behind," etc.)

48-60m

› Describe and compare shapes using their attributes in their home language(s) (e.g., "a triangle has three sides, but a square has four.")

› Combine and separate shapes to make other shapes (e.g., using two triangles to make a square)

› Build more complex models of buildings, structures, or areas (e.g., their classroom or play-ground) with three-dimensional shapes, such as building blocks

› Correctly name familiar shapes in their home language(s) (e.g., circle, triangle, and square) and less familiar shapes (e.g., hexagon, trapezoid, and rhombus)

› Correctly name some three-dimensional shapes in their home language(s) (e.g., cube, cone, cylinder)

› Understand and use language or similar words in their home language(s) related to directionality, order, and the position of objects, such as "up," "down," "in front," and "behind"